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Spectral Analysis Of ICA Decomposition Of Linear Mixture Of Sine Waves

Introduction

The objective of this set of experiments is to verify that simulated noise, superimposed upon a linear mixture of sine waves of different frequencies, can, through ICA, be represented by a single independent component (IC).

We will demonstrate this by comparing the spectra of the added noise (n) to the spectra of the independent component that most closely represents this noise, as determined by its correlation to the noise template.

Furthermore, we wish to demonstrate that it may be possible to enhance the effectiveness of this noise representation by frequency filtering this independent component to remove harmonics that do not correspond to the added noise and thus are more properly contained within other independent components. This subsequent post-processing should result in a 'clean' IC, representing only the noise, and subsequently allow the noise to be surgically removed from the data.

Description Of The Experiment

The sine waves (s) are linearly mixed by left multiplying the vector of sinusoids by a mixing matrix, A:

$$x = A * s$$

where:

- $x = [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5]^T$
- $s = [\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4, \mathbf{s}_5]^T$
- $A = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5]$

Noise is then projected onto the mixture through the use of a noise template:

$$x_{Noise} = A * s + Template_{Noise} * n$$

After applying ICA to the noisy data, x_{Noise}, the noise template and noise are now ideally contained as components of a new mixing matrix and a new vector of independent components, respectfully:

$$x_{Noise} = A' * s'$$

where:

- $s' = [n, \mathbf{s}_2', \mathbf{s}_3', \mathbf{s}_4', \mathbf{s}_5']$
- $A' = [Template_{Noise}, \mathbf{a}_2', \mathbf{a}_3', \mathbf{a}_4', \mathbf{a}_5']$

Experimental Parameters

Sine Waves (s):

- 5 x 2500 Matrix (5 channels, 2500 samples)
- s_1: 0.010Hz; s_2: 0.016Hz; s_3: 0.022Hz; s_4: 0.028Hz; s_5: 0.034Hz

Noise (n):

- $Template_{Noise} = [0.750, 1.000, 0.750, 0.250, 0.125]^T$
- 1 x 2500 Row Vector (1 channel, 2500 samples) ; 0.004Hz

Mixing Matrix (A):

- 5 x 5, generated randomly via MATLAB function randn(5)

FastICA runs used TanH contrast function and random starting seed.

Discussion of Results

Spectral Analysis of Independent Components and Misallocation of Variance

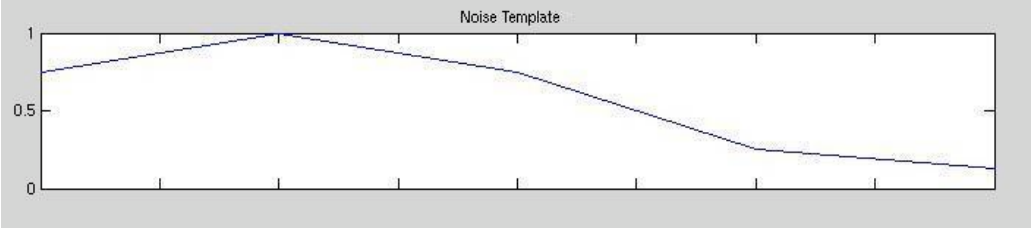
A spectral analysis of the independent components that correspond to the noise template with a correlation >= 0.90 revealed contamination with harmonics from non-noise related sources (Figure Set # 1 for FastICA and Figure Set # 2 for InfoMax).

Furthermore, based upon their harmonic composition, the independent components with the greatest noise template correlation may not be the best match for the noise activity (Compare Figure Set # 3 to # 4 for FastICA and Figure Set # 5 to # 6 for InfoMax). Specifically, independent components with a lesser correlation to the noise template were, in some cases, a better match to the noise activity under a metric comparing their harmonic components to the harmonic composition of the added noise.

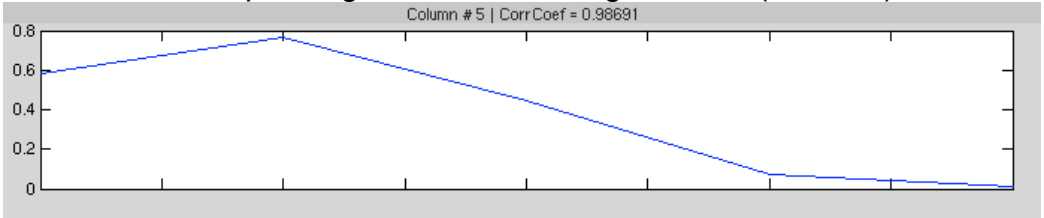
This suggests that noise template correlation, in and of itself, may not be a sufficient metric for measuring the effectiveness of the ICA process and cleaning the data of noise contamination. However, the data suggest that some post-processing of the independent components may improve the situation. For example, by applying a notch filter, centered upon the fundamental harmonic of the noise, to the independent components determined by the noise template correlation, it may be possible to filter out the contributions from harmonics more properly contained within other independent components.

Fig. Set # 1: (FastICA) Noise Template Correlation = 0.9869

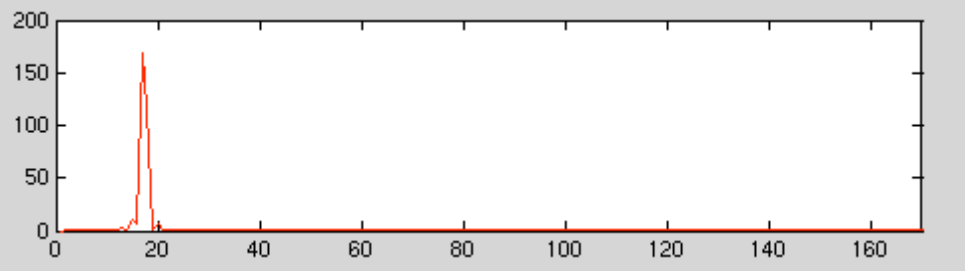
Noise Template



Corresponding Column Of Mixing Matrix A (FastICA)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC

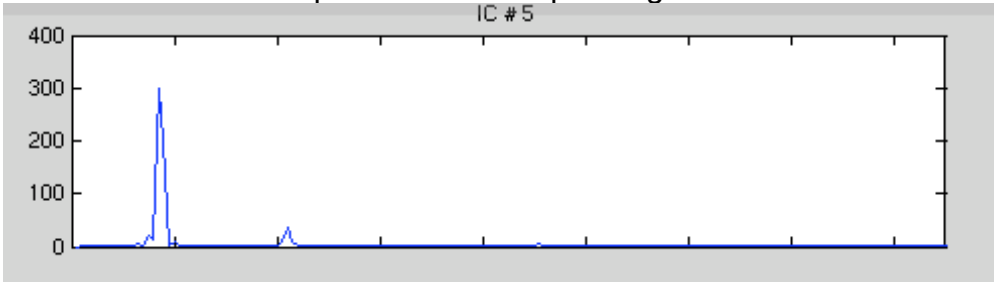
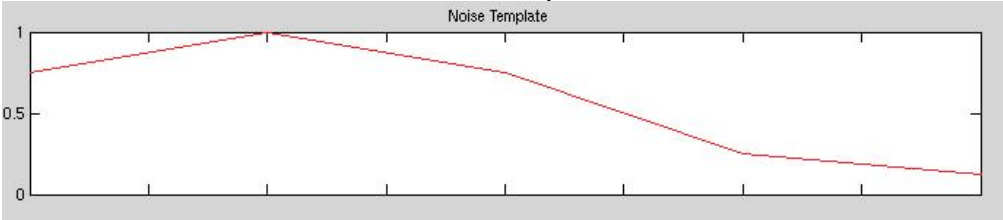
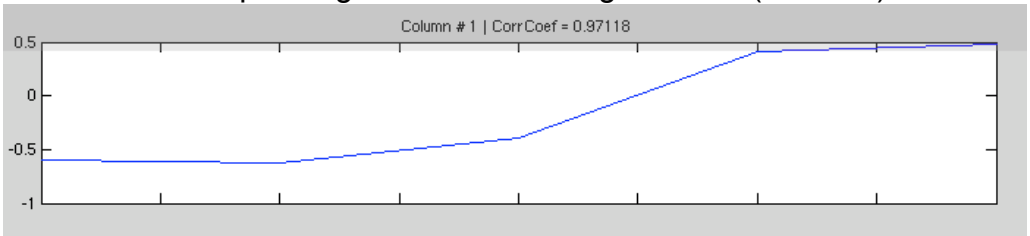


Fig. Set # 2: (InfoMax) Noise Template Correlation = 0.9712

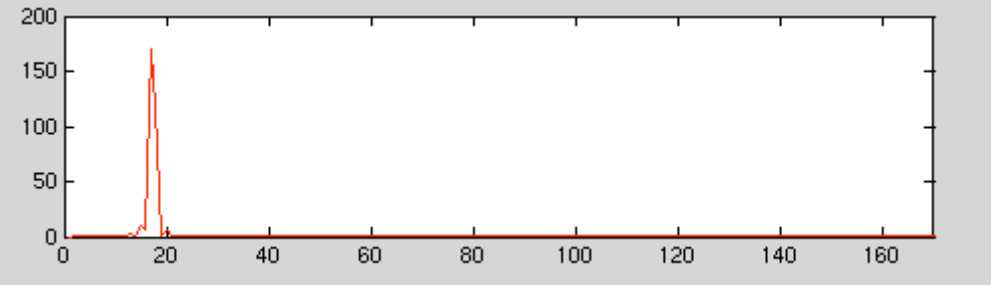
Noise Template



Corresponding Column Of Mixing Matrix A (InfoMax)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC

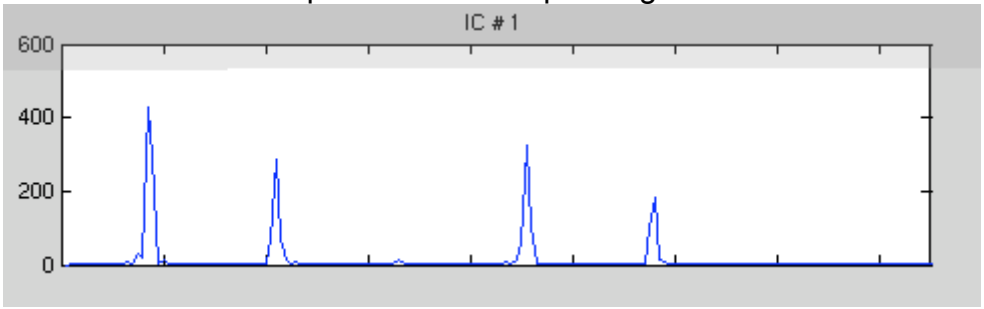
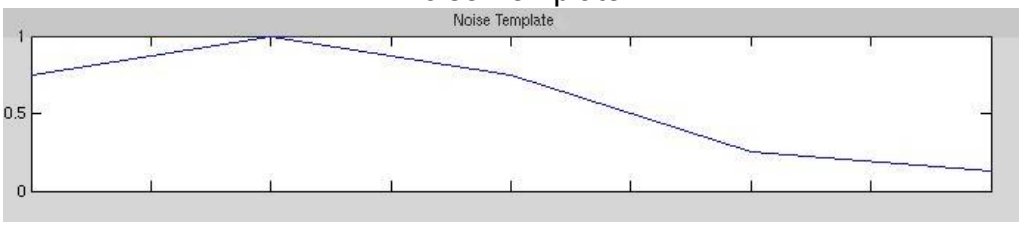
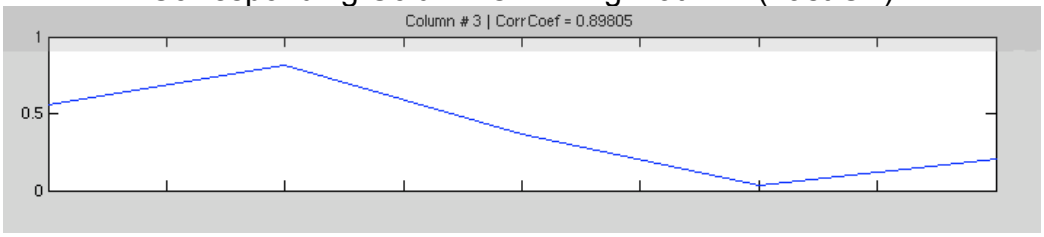


Fig. Set # 3: (FastICA) Noise Template Correlation = 0.8981

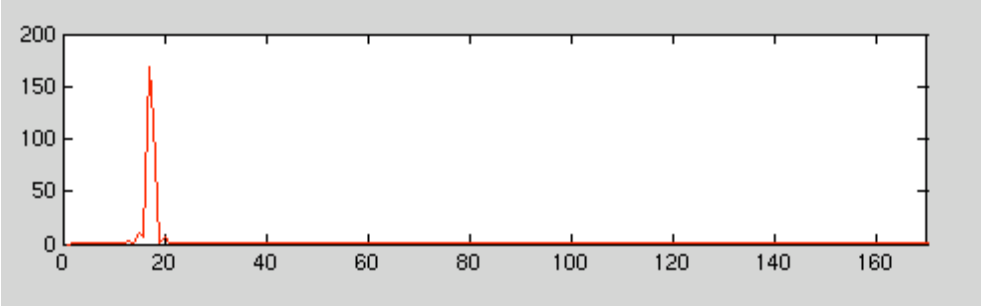
Noise Template



Corresponding Column Of Mixing Matrix A (FastICA)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC

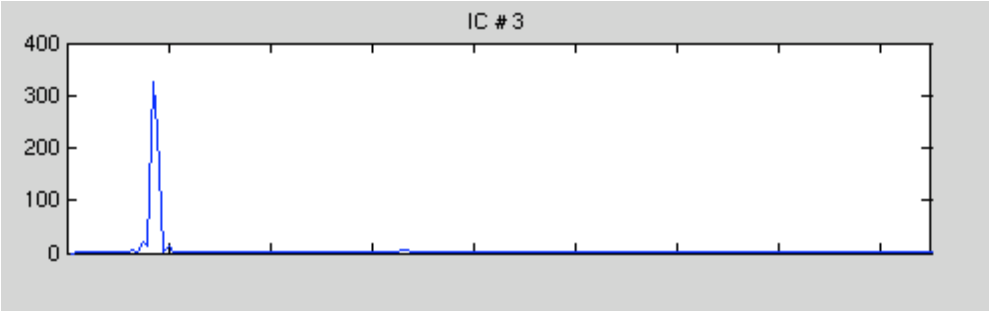
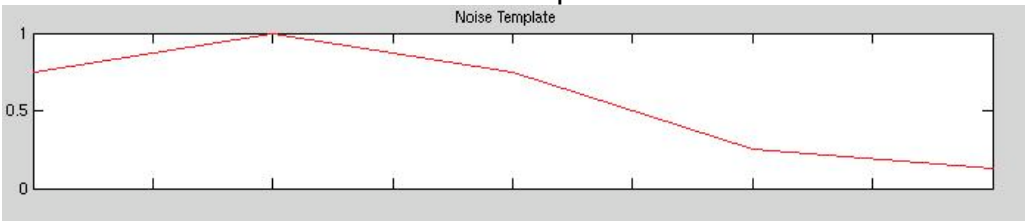
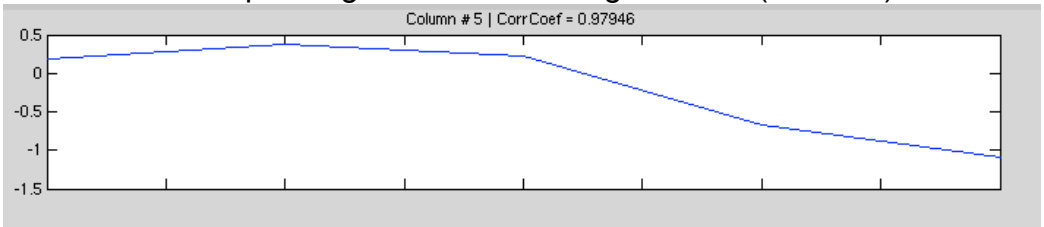


Fig. Set # 4: (FastICA) Noise Template Correlation = 0.9795

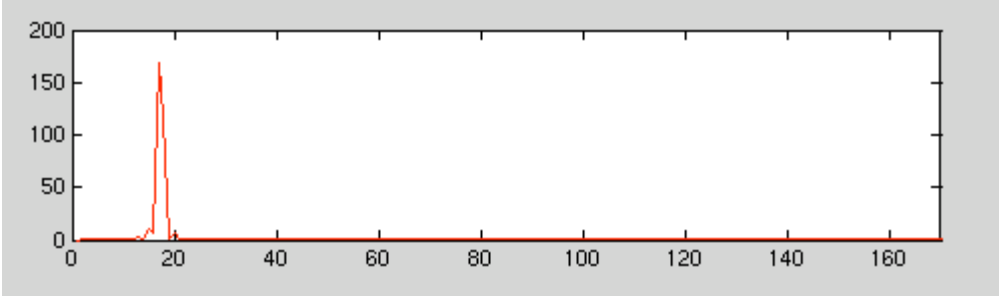
Noise Template



Corresponding Column Of Mixing Matrix A (FastICA)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC

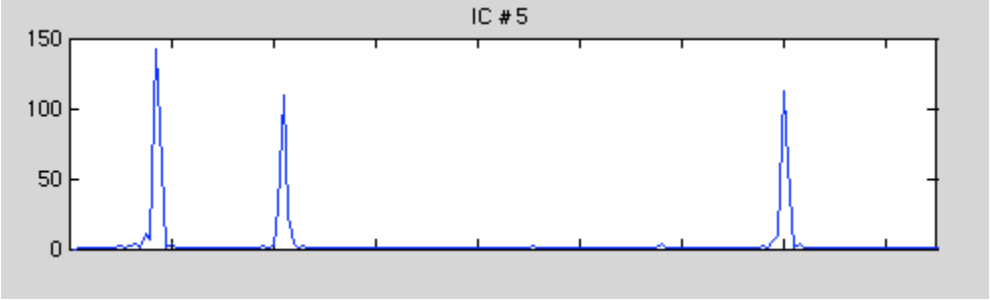
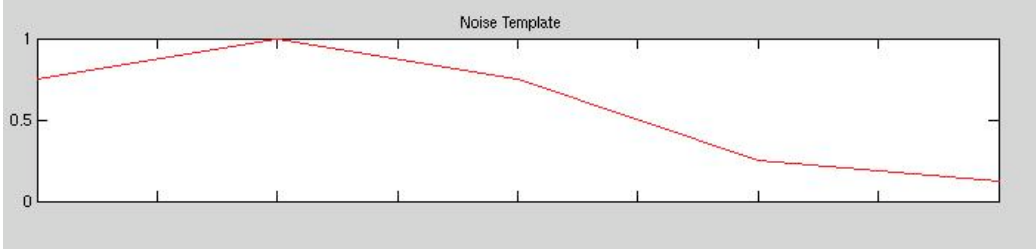
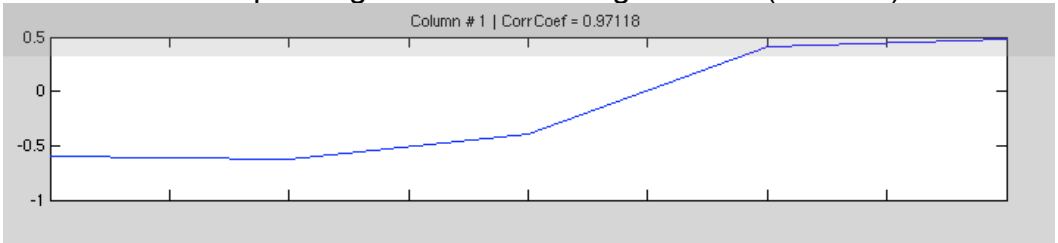


Fig. Set # 5: (InfoMax) Noise Template Correlation = 0.9712

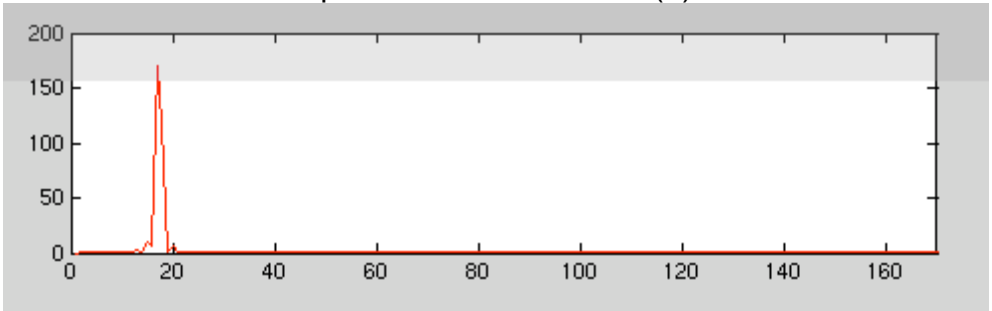
Noise Template



Corresponding Column Of Mixing Matrix A (InfoMax)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC

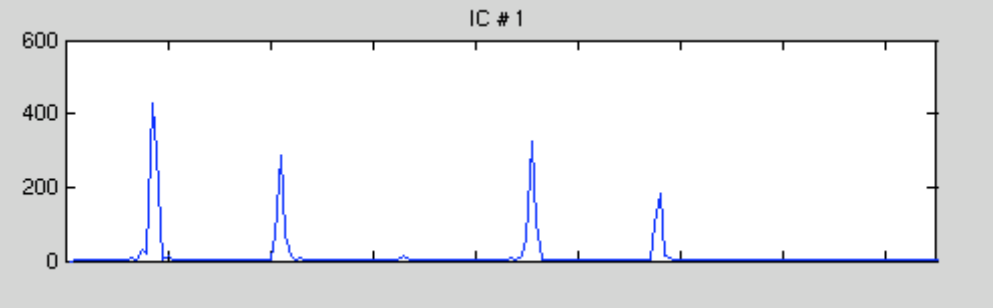
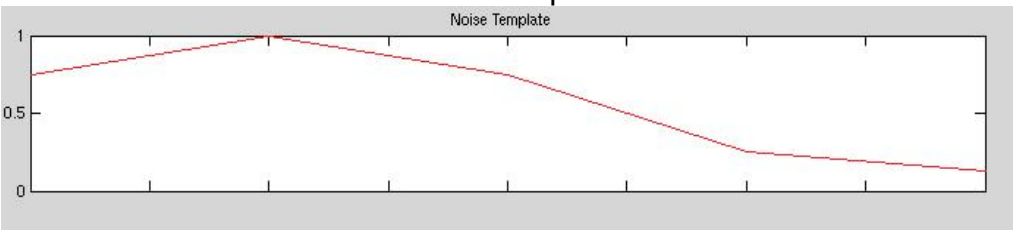
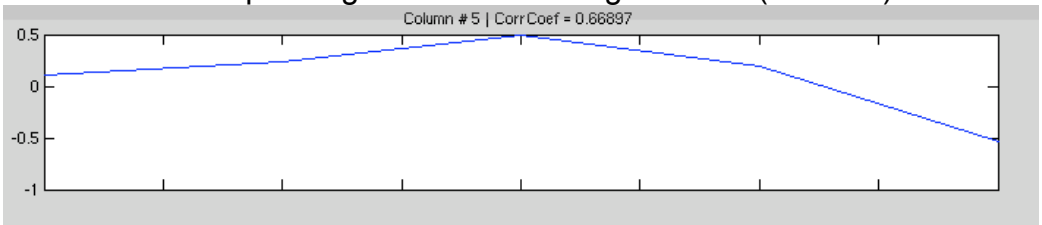


Fig. Set # 6: (InfoMax) Noise Template Correlation = 0.6690

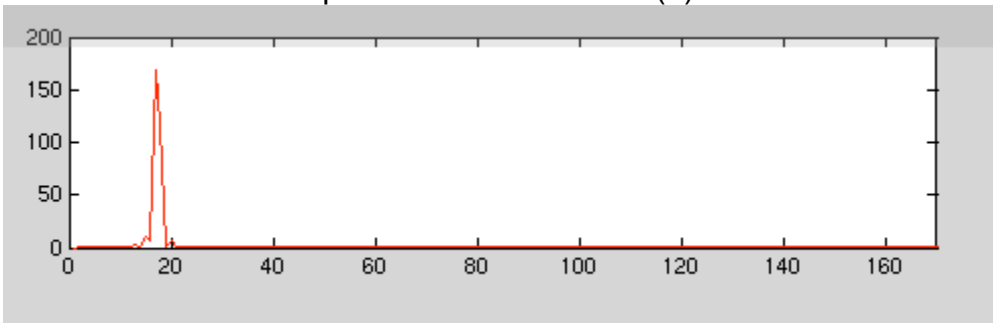
Noise Template



Corresponding Column Of Mixing Matrix A (InfoMax)



Spectra Of Added Noise (n)



Spectra Of Corresponding IC